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Department

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Course List
List Of Courses for Aerospace Engineering

Course Code	Course Name	Faculty
AE651	Aerodynamics of Compressors and Turbines	Prof Bhaskar Roy

List of Courses

Course List for Electrical Engineering
List Of Courses For Electrical Engineering

Course Code	Course Name	Faculty
<u>EE111</u>	Introduction to Electrical Systems	Prof.B.G.Fernandes
EE603	Digital Signal Processing and its Applications	Prof. Vikram M. Gadre

Cours list for Electrical Engineering

EE111-Syllabus
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Syllabus

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EE111-Lecture16

Subject :EE111 Lecture no.:16 Lecture By : PROF .B.G. Fernandes
Department Of Electrical Engineering, IIT Bombay Topic : Mutually coupled circuits Video File Reference : EE111- - L16

I intend to complete this chapter Mutually Coupled Circuits.. (Refer slide time: 00:17 – 00:18)

Sub-Topic:

- Mutually coupled circuits (contd..)

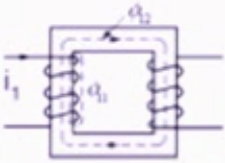
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Slide 1 Mutually coupled circuits

(Refer slide time: 00:28 – 01:35)

REVIEW

- Mutual inductance, $M = N_2 \frac{d\phi_{12}}{di_1}$
 $= N_1 \frac{d\phi_{21}}{di_2}$



- Parameter relating the 'V' in one circuit due to time varying i_1 flowing in another circuit.

$$\therefore M = K \sqrt{L_1 L_2}$$
$$k \leq 1$$

↳ coefficient of coupling

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Slide 2 Mutual Inductance

In the last class we defined the term mutual inductance. It is defined as or it relates the voltage induced in one coil due to the time varying current flow in the other circuit. Mutual inductance is given by, $M = N_2 \times d \varphi_{12} / di_1$, φ_{12} is produced by i_1 that is flowing in coil 1. This also can be defined as $N_1 \times d \varphi_{21} / di_2$, φ_{21} is the part of the flux linking coil 1 and the current that flowing in coil 2 is i_2 . Now what is the relationship between the mutual inductance and the self inductance of two coils? This is given by $k \sqrt{L_1 L_2}$, where k is the coefficient of coupling. If coils are tightly coupled, $k \leq 1$. (Refer slide time: 01:35– 02:58)

Sign convention:

Case 1: Fluxes are additive:

Total flux linking coil 1 = $N_1 (\phi_1 + \phi_2)$

$$v_1 = L_1 \frac{di_1}{dt} + M \frac{di_2}{dt}$$

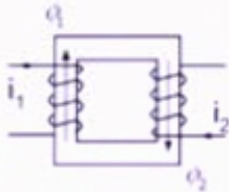
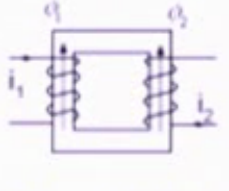
$$\bar{V}_1 = j\omega (L_1 \bar{I}_1 + M \bar{I}_2)$$

Similarly, $\bar{V}_2 = j\omega (L_2 \bar{I}_2 + M \bar{I}_1)$

Case 2: Opposing fluxes:

Total flux linking coil 1 = $N_1 (\phi_1 - \phi_2)$

$$\therefore v_1 = L_1 \frac{di_1}{dt} - M \frac{di_2}{dt} \quad \bar{V}_1 = j\omega (L_1 \bar{I}_1 - M \bar{I}_2) \quad \& \quad \bar{V}_2 = j\omega (L_2 \bar{I}_2 - M \bar{I}_1)$$

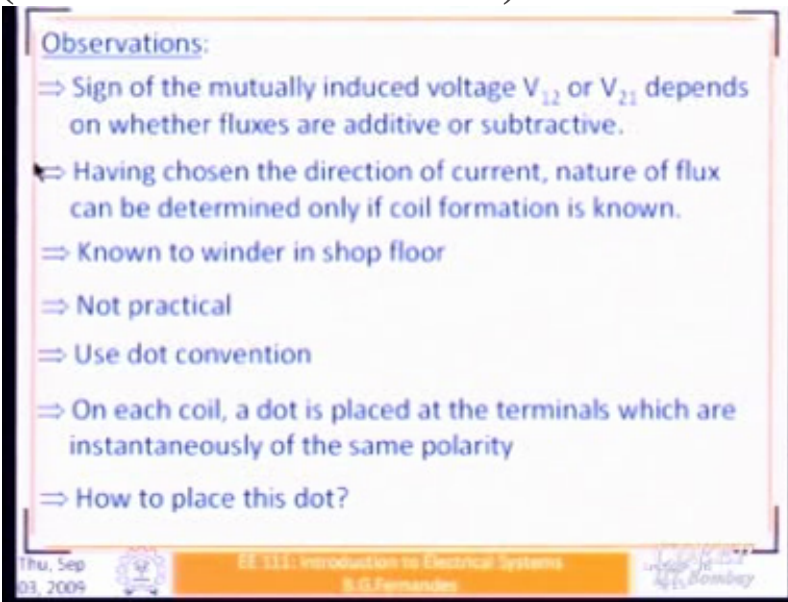



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Slide 3 Sign Convention

Sign Convention. There are two coils, flux produced by these two coils can either add or oppose. What happens if they add? What is the net flux link in the coil 1, if the fluxes are additive, φ_1 is the flux produced by coil 1, φ_2 is the flux produced by coil 2. They are additive so net flux link by coil 1 is $\varphi_1 + \varphi_2$. What is the voltage equation if I neglect the winding terminal voltage? It should be equal to the induced voltage and that induced voltage is equal to the rate of change of flux $v_1 = L_1 \times di_1 / dt + M \times di_2 / dt$. So in frequency domain the RMS value of $v_1 = j\omega(L_1 I_1 + M I_2)$. Similarly $v_2 = j\omega(L_2 I_2 + M I_1)$. If they are opposing then what happens, it is $L_1 I_1 - M I_2$.

(Refer slide time: 02:59– 03:14)



Observations:

- ⇒ Sign of the mutually induced voltage V_{12} or V_{21} depends on whether fluxes are additive or subtractive.
- ⇒ Having chosen the direction of current, nature of flux can be determined only if coil formation is known.
- ⇒ Known to winder in shop floor
- ⇒ Not practical
- ⇒ Use dot convention
- ⇒ On each coil, a dot is placed at the terminals which are instantaneously of the same polarity
- ⇒ How to place this dot?

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Slide 4 Observations

So what are the observations, the sign of mutually induced voltage v_{12} or v_{21} depends on whether fluxes are additive or subtractive. But then having chosen the direction of current how do I determine the flux if I do not know the way that it is wound, how do I determine the direction of flux? I have to assume the direction of current, then I need to know the way the coil is wound, who knows about the way that it is wound. The winder in the shop floor. Having designed the coil, having fabricated you get a black box can I know how the coil is wound? It is just not possible, if that is the case how do I write the voltage equations? (Refer slide time: 03:50– 04:30)

The sign of mutual induced voltage v_1 and v_2 depends on whether fluxes are additive or subtractive. (Refer slide time: 03:57– 04:23)

Sign convention:

Case 1: Fluxes are additive:

Total flux linking coil 1 = $N_1 (\phi_1 + \phi_2)$

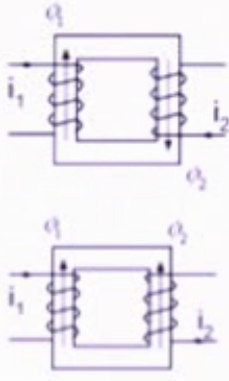
$$v_1 = L_1 \frac{di_1}{dt} + M \frac{di_2}{dt}$$

$$\bar{V}_1 = j\omega (L_1 \bar{I}_1 + M \bar{I}_2)$$

Similarly, $\bar{V}_2 = j\omega (L_2 \bar{I}_2 + M \bar{I}_1)$

Case 2: Opposing fluxes:

Total flux linking coil 1 = $N_1 (\phi_1 - \phi_2)$

$$\therefore v_1 = L_1 \frac{di_1}{dt} - M \frac{di_2}{dt} \quad \bar{V}_1 = j\omega (L_1 \bar{I}_1 - M \bar{I}_2) \quad \& \quad \bar{V}_2 = j\omega (L_2 \bar{I}_2 - M \bar{I}_1)$$


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Slide 5 Sign convention

How do I determine that the fluxes are additive or subtractive? I can assume the direction of current, either you can enter the terminal or leave. Having assumed the direction of current I need to know the way that the coil is wound, that information was only known to the person who winds the coil, if that information is not known how do I write the voltage equations?
Using dot conventions. (Refer slide time: 04:24– 04:45)

Observations:

- ⇒ Sign of the mutually induced voltage V_{12} or V_{21} depends on whether fluxes are additive or subtractive.
- ⇒ Having chosen the direction of current, nature of flux can be determined only if coil formation is known.
- ⇒ Known to winder in shop floor
- ⇒ Not practical
- ⇒ Use dot convention
- ⇒ On each coil, a dot is placed at the terminals which are instantaneously of the same polarity
- ⇒ How to place this dot?

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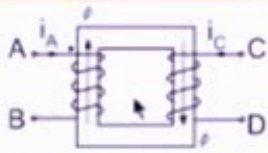
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Slide 6 Observations

For each coil the dot is placed at the terminals which are instantaneously of the same polarity. The dot convention, the two dots are placed at two different or two different coils. What did they imply? At any given time these two terminals are of the same polarity. The cursor question now is how to place this dot? (Refer slide time: 04:54– 05:04)

- Assume that physical arrangement and mode of each winding are known
- Select any one terminal (say A) of one coil and place a dot
- Assign the current into the dot (say i_A)
- Use R.H. rule to determine the direction of the magnetic field produced by i_A (say ϕ_A)
- Pick one terminal of second coil (say 'C') and assign a current into this terminal (say i_C)
- Use R.H. rule to determine the direction of flux produced by i_C (say ϕ_C)



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Slide 7 Observations

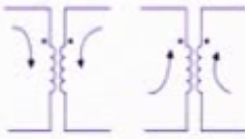
Two mutually coupled coil, assume that the physical arrangement and mode of each winding is known. The first time it is known, so take any terminal, say A and I will place the dot there and will assign the current into the dot, use the right hand rule to determine the direction of the flux. Pick one terminal of the second coil, (Refer slide time: 05:20– 05:43)

any one of the terminal and assign the current into the terminal. I have taken C and assigned the current into the terminal determine the direction of the flux. (Refer slide time: 05:44– 06:33)

- Compare the directions of ϕ_A and ϕ_C . If the fluxes are additive, place the dot on the terminal of the second coil where the current i_C enters (C in this case). If the fluxes are subtractive, place the dot on the terminal of the second coil where the current leaves.

Sign of 'V' of M in mesh current equations:

- When both assumed currents enter or leave a pair of coupled coils at the dotted terminals, the sign of M terms will be the same as the sign of the L terms.



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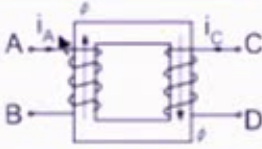
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Slide 8 Sign of V of M in mesh current eqations

If the flux are ready to place the dot on the terminal of the second coil where current i_C enters else place the dot in the other terminal. (Refer slide time: 06:34– 06:40)

- Assume that physical arrangement and mode of each winding are known
- Select any one terminal (say A) of one coil and place a dot
- Assign the current into the dot (say i_A)
- Use R.H. rule to determine the direction of the magnetic field produced by i_A (say ϕ_A)
- Pick one terminal of second coil (say 'C') and assign a current into this terminal (say i_C)
- Use R.H. rule to determine the direction of flux produced by i_C (say ϕ_C)



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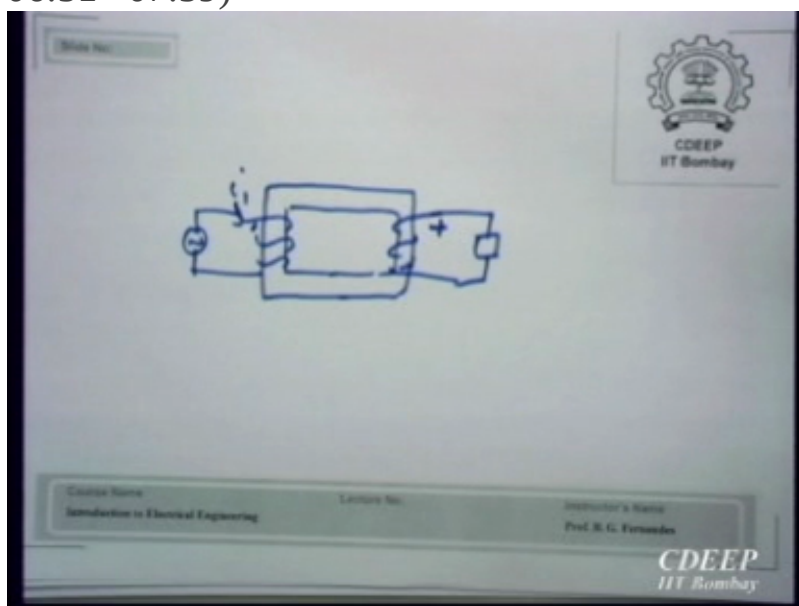
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Slide 9 Observations

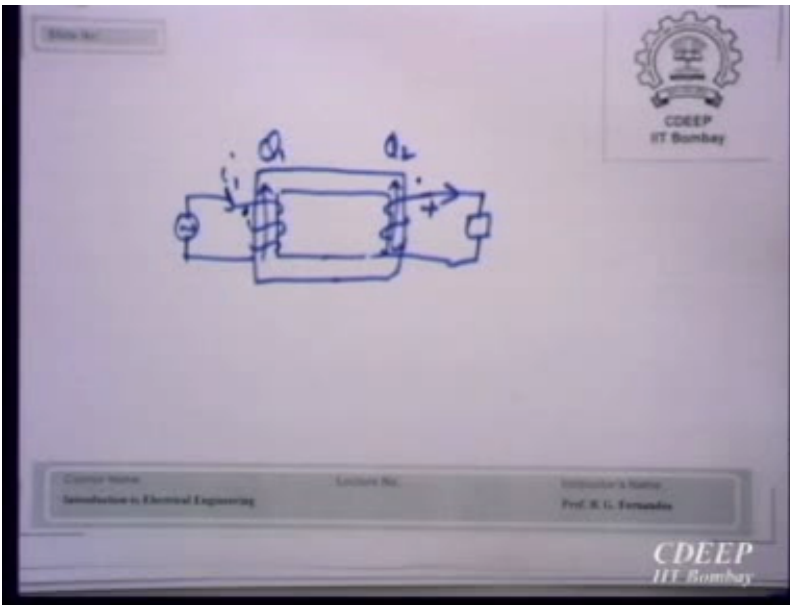
Take any one coil, any terminal place, a dot there assign the current determine the direction of current. Take the second coil, any one terminal

don't place the dot now you assign the current into the terminal, determine the direction of flux. If the fluxes are additive, place the dot at the terminal where the current enters else place the dot in the other terminal. So at any given time these two terminals are of the same polarity. i_A plus then i_C also plus, why only if the flux are additive this should be positive that is the question being asked. I will answer this question now. (Refer slide time: 06:51– 07:59)



Slide 10 Coil

i_1 is AC so, the flux produced by i_1 is alternating. It will link the coil to the voltage induced. If I connect a load, current will flow, that current will produce its own flux in the coil 2. This is the positive terminal current entering the node, at any given time. I assume that this is positive, current is entering that node. If I assume this to be positive at a given time, what should be the direction of the current? I am connecting the load across the coil, the voltage is induced in the coil 2. At some time the polarity of the terminal will be positive now what should be the direction of that current at that time, it should enter this terminal, what should be the direction of the flux produced by the second coil? (Refer slide time: 08:05– 08:18)



Slide 11 Coil

If this is the direction of the flux forced by ϕ_1 and this is also the flux produced by the ϕ_2 . What did I do here, I assigned I took one terminal and in the second coil I assigned the current entering the terminal and I saw if that fluxes are additive then dot remains there. If the flux are opposing, the dot goes to second terminal, both are matching isn't it. (Refer slide time: 08:41– 08:52)

- Compare the directions of ϕ_A and ϕ_C . If the fluxes are additive, place the dot on the terminal of the second coil where the current i_C enters (C in this case). If the fluxes are subtractive, place the dot on the terminal of the second coil where the current leaves.

Sign of 'V' of M in mesh current equations:

- When both assumed currents enter or leave a pair of coupled coils at the dotted terminals, the sign of M terms will be the same as the sign of the L terms.

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- Compare the directions of ϕ_A and ϕ_C . If the fluxes are additive, place the dot on the terminal of the second coil where the current i_C enters (C in this case). If the fluxes are subtractive, place the dot on the terminal of the second coil where the current leaves.

Sign of 'V' of M in mesh current equations:

- When both assumed currents enter or leave a pair of coupled coils at the dotted terminals, the sign of M terms will be the same as the sign of the L terms.



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Slide 12 Sign of V of M in mesh current equations

Writing the voltage equation in the mutually coupled coil what should be the sign of L and M? (Refer slide time: 08:52– 09:05)

Sign convention:

Case 1: Fluxes are additive:

Total flux linking coil 1 = $N_1 (\phi_1 + \phi_2)$

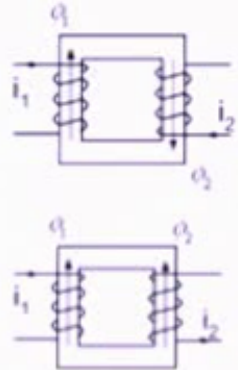
$$v_1 = L_1 \frac{di_1}{dt} + M \frac{di_2}{dt}$$

$$\bar{V}_1 = j\omega (L_1 \bar{I}_1 + M \bar{I}_2)$$

Similarly, $V_2 = j\omega (L_2 \bar{I}_2 + M \bar{I}_1)$

Case 2: Opposing fluxes:

Total flux linking coil 1 = $N_1 (\phi_1 - \phi_2)$

$$\therefore v_1 = L_1 \frac{di_1}{dt} - M \frac{di_2}{dt} \quad V_1 = j\omega (L_1 \bar{I}_1 - M \bar{I}_2) \quad \& \quad V_2 = j\omega (L_2 \bar{I}_2 - M \bar{I}_1)$$


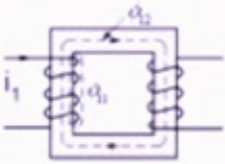
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Slide 13 Sign convention

We assume the direction of current and we knew the way the coil is wound. We determine the direction of flux if they are additive, we find a sign of given v_{12} and v_{21} are the same. (Refer slide time: 09:05– 09:17)

REVIEW

- Mutual inductance, $M = N_2 \frac{d\phi_{12}}{di_1}$
 $= N_1 \frac{d\phi_{21}}{di_2}$



- Parameter relating the 'V' in one circuit due to time varying i_1 flowing in another circuit.

$$\therefore M = K \sqrt{L_1 L_2}$$

$$k \leq 1$$

↳ coefficient of coupling

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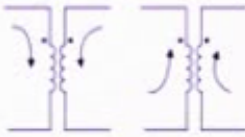
Slide 14 Mutual Inductance

Now the second problem is that how you know the way the coil is wound?
 How did you solve that problem, by placing a dot? (Refer slide time: 09:17– 09:56)

- Compare the directions of ϕ_A and ϕ_C . If the fluxes are additive, place the dot on the terminal of the second coil where the current i_C enters (C in this case). If the fluxes are subtractive, place the dot on the terminal of the second coil where the current leaves.

Sign of 'V' of M in mesh current equations:

- When both assumed currents enter or leave a pair of coupled coils at the dotted terminals, the sign of M terms will be the same as the sign of the L terms.



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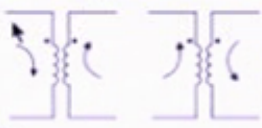
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Slide 15 Sign of V of M in mesh current equations

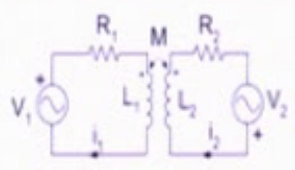
Now we know that here is a mutually coupled circuit, dots have been placed. What should be the sign of L and M terms, if the fluxes are additive, sign of L is same as sign of M. When the fluxes are additive, current entering the dot terms or the current leaving they are opposite. (Refer slide time: 09:57– 10:09)

• If one current enters at a dotted terminal and one leaves by a dotted terminal, sign of M terms are opposite to the sign of L terms



e.g. i_1 enters the dot & i_2 leaves the dot

'V' in $L_1 = j\omega [L_1 \bar{I}_1 - M \bar{I}_2]$
 'V' in $L_2 = j\omega [L_2 \bar{I}_2 - M \bar{I}_1]$
 $\therefore V_1 = R_1 \bar{I}_1 + j\omega [L_1 \bar{I}_1 - M \bar{I}_2]$
 $V_2 = R_2 \bar{I}_2 + j\omega [L_2 \bar{I}_2 - M \bar{I}_1]$
 $\bar{V}_1 = (R_1 + j\omega L_1) \bar{I}_1 - j\omega M \bar{I}_2 \quad \bar{V}_2 = -j\omega M \bar{I}_1 + (R_2 + j\omega L_2) \bar{I}_2$



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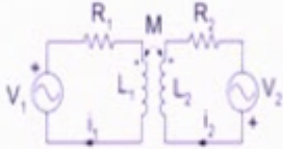
Slide 16 equations

If one coil current enters the dot and the other coil leaves the dot, if one current enters at a dotted terminal and one leaves by a dotted terminal sign of M terms are opposite to the sign of L terms. Fluxes are additive, then total flux linkage is $N_1 \times \varphi_1 + \varphi_{21}$. (Refer slide time: 10:20– 10:42)

- If one current enters at a dotted terminal and one leaves by a dotted terminal, sign of M terms are opposite to the sign of L terms

e.g. i_1 enters the dot & i_2 leaves the dot

'V' in $L_1 = j\omega [L_1 i_1 - M i_2]$
 'V' in $L_2 = j\omega [L_2 i_2 - M i_1]$
 $\therefore V_1 = R_1 i_1 + j\omega [L_1 i_1 - M i_2]$
 $V_2 = R_2 i_2 + j\omega [L_2 i_2 - M i_1]$
 $V_1 = (R_1 + j\omega L_1) i_1 - j\omega M i_2$ $V_2 = -j\omega M i_1 + (R_2 + j\omega L_2) i_2$



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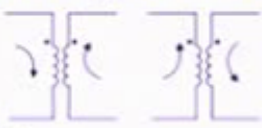
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Slide 17 equations


Now let us see about the mutually coupled coil, let us see the voltage equation you assign the current the way you own either clock wise or anti clock wise. Now, what is the voltage equation? see what happens in coil 1, current enters the dot what about in coil 2 it leaves the dot terminal or what is the voltage induced in L_1 there are two mutually coupled coil, both of them are carrying the current. So, voltage induced in one coil, it has two components what are they? One is due to its own flux and the other one is due to the flux produced by the second coil. Now, the sign of L and M depends upon the direction of current the we found that in this case, in one case enters the dot in second loop it leaves the dot. (Refer slide time:11:12 to 12:12)

- If one current enters at a dotted terminal and one leaves by a dotted terminal, sign of M terms are opposite to the sign of L terms



e.g. i_1 enters the dot & i_2 leaves the dot

'V' in $L_1 = j\omega[L_1 I_1 - M I_2]$
 'V' in $L_2 = j\omega[L_2 I_2 - M I_1]$
 $\therefore V_1 = R_1 I_1 + j\omega[L_1 I_1 - M I_2]$
 $V_2 = R_2 I_2 + j\omega[L_2 I_2 - M I_1]$
 $\bar{V}_1 = (R_1 + j\omega L_1) \bar{I}_1 - j\omega M \bar{I}_2$ $\bar{V}_2 = -j\omega M \bar{I}_1 + (R_2 + j\omega L_2) \bar{I}_2$



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Slide 18 equations

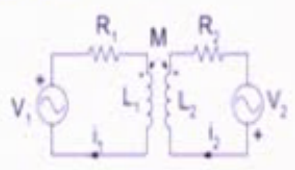
So, what is the voltage induced in $L_1 = j\omega[L_1 I_1 I_1 - M I_2]$ Voltage induced in second coil is $j\omega[L_2 I_2 - M I_1]$ therefore $V_1 = R_1 I_2 + j\omega[L_2 I_2 - M I_2]$. The second coil voltage equation $V_2 = R_2 I_2 + j\omega[L_2 I_2 - M I_1]$. First step determine the currents and find out whether it is entering the dot or leaving the dot that is the step number one assign the mesh current or mesh current in the loops find out whether the currents are current in respective business entering the dot or leaving dot. First step that has to be done then you write the voltage induce in individual coils and finally you write mesh equation. These are the three steps to be done. (Refer slide time:12:34 to 12.57)

- If one current enters at a dotted terminal and one leaves by a dotted terminal, sign of M terms are opposite to the sign of L terms



e.g. i_1 enters the dot & i_2 leaves the dot

'V' in $L_1 = j\omega [L_1 I_1 - M I_2]$
 'V' in $L_2 = j\omega [L_2 I_2 - M I_1]$
 $\therefore V_1 = R_1 I_1 + j\omega [L_1 I_1 - M I_2]$
 $V_2 = R_2 I_2 + j\omega [L_2 I_2 - M I_1]$
 $V_1 = (R_1 + j\omega L_1) I_1 - j\omega M I_2$ $V_2 = -j\omega M I_1 + (R_2 + j\omega L_2) I_2$



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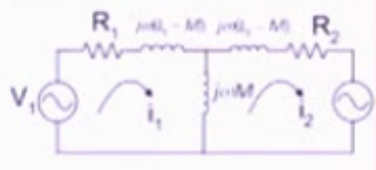
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Slide 19 equations

So v_1 is equal to v_1 is given by this equation v_2 and write in matrix form, first one is $(R_1 + j\omega L_1) I_1 - j\omega M I_2$. Second term, second row, first element, $-j\omega M I_1 + (R_2 + j\omega L_2) I_2$. Magnetic coupling exist between these two coils are electrically they are isolated by seeing this equations can I write (Refer slide time:13:11 to 14:37)

$$\begin{bmatrix} R_1 + j\omega L_1 & -j\omega M \\ -j\omega M & R_2 + j\omega L_2 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

\therefore Conductively coupled equivalent circuit

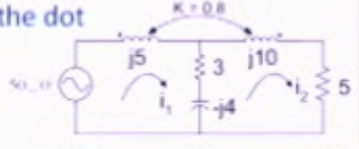


e.g.

$jX_M = jK\sqrt{X_1 X_2}$
 $= j0.8\sqrt{50} = j5.66$

i_1 enters the dot and i_2 leaves the dot

'V' in $j5 = j5I_1 - j5.66I_2$
 'V' in $j10 = j10I_2 - j5.66I_1$



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Slide 20 Conductively coupled equivalent circuit

voltage equation for can I draw the circuit which is conductively couple this is the voltage and current equation from these equations can I draw conductively couple circuit. See what is this term; second term first row first term. sum of all, impedance of the mesh 1 second row second element sum of all impedance connect to mesh 2, these two are common. Write the voltage equation here what do I get $R_1 + j\omega L_1 + j\omega M$ this is the mutual term common for 1 and 2, this is sum of all what do I call this as L_1 is the self inductance, M is the mutual inductance what could be the difference L_1 and M , what is $L_1 - M$, what it could be? Self flux minus the mutual, what is left now? Leakage. $\Phi_1 = \Phi_{11} + \Phi_{12}$. Φ_{12} is the mutual, Φ_{11} means either part off coil one leakage. (Refer slide time:14:44 to 16:07)

$$\begin{bmatrix} R_1 + j\omega L_1 & -j\omega M \\ -j\omega M & R_2 + j\omega L_2 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

∴ Conductively coupled equivalent circuit

e.g.

$$jX_M = jK\sqrt{X_1 X_2}$$

$$= j0.8\sqrt{50} = j5.66$$

i_1 enters the dot and i_2 leaves the dot

'V' in $j5 = j5I_1 - j5.66I_2$

'V' in $j10 = j10I_2 - j5.66I_1$

The slide also contains two circuit diagrams. The top diagram shows a general conductively coupled equivalent circuit with two meshes. The bottom diagram shows a specific circuit with a voltage source V_1 , a resistor R_1 , and an inductor $j5$ in the first mesh. The second mesh contains a resistor R_2 , an inductor $j10$, and a resistor 5 . A mutual inductance $j5.66$ is indicated between the two meshes.

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Slide 21 Conductively coupled equivalent circuit

This is common for mutual. I will just solve another problem, say the mutually coupled circuit. coefficient of coupling is given as .8, L_1 is 5, j is 10. $jX_M = \sqrt{j5.66}$. Now, I will assign the current i_1 in mesh 1, i_2 in mesh 2. i_1 is the current flowing in $j5$ entering the nod, i_2 is the current flowing in $j10$ leaving the dot. In first, it enters the dot in $j10$ it leaves the dot. What should be the voltage induce in $j5$ if there is no coupling, $j5i_1 - j5.66i_2$, voltage induce in $j10$, $j10i_2 - j5.66i_1$. (Refer slide time:16:08 to 17:05)

Mesh 1:

$$50 \angle 0 = (3-j4)(i_1 - i_2) + j5i_1 - j5.66i_2$$

$$= (3+j5-j4)i_1 - (3-j4+j5.66)i_2$$

$$= (3+j1)i_1 - (3+j1.66)i_2$$

Mesh 2:

$$0 = 5i_2 + j10i_2 - j5.66i_1 + (3-j4)(i_2 - i_1)$$

$$(-3+j4-j5.66)i_1 + (5+j10+3-j4)i_2 = 0$$

$$(-3-j1.66)i_1 + (8+j6)i_2 = 0$$

2 equations, 2 unknowns, solve for \bar{I}_1 & \bar{I}_2

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Slide 22 Mesh 1 and Mesh 2 quations

Voltage equation 50 is equal to voltage induce in j5 plus voltage across this branch. What is the voltage drop in this branch $(3-j4)(i_1-i_2)+j5i_1-j5.66i_2$. Similarly in mesh 2, $5i_2+(3-j4)(i_2-i_1)$ plus the drop across j10 so there are two equation solve for i_1 and i_2 . (Refer slide time:17:06 to 18:55)

Replacing series connected mutually coupled coils:

- Place the dots
- Current enters both the dots
- Sign of 'M' is the same as L

'V' in $L_1 = j\omega L_1 i_1 + j\omega M i_2$

$$= j\omega i_1 (L_1 + M)$$

'V' in $L_2 = j\omega L_2 i_2 + j\omega M i_1$

$$= j\omega i_2 (L_2 + M)$$

$\therefore V = i_1 R + j\omega i_1 (L_1 + M) + j\omega i_2 (L_2 + M)$

$$= i_1 R + j\omega i_1 (L_1 + L_2 + 2M)$$

$$= i_1 R + j\omega i_1 L_{eq}$$

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Slide 23 Replacing series connected mutually coupled coils

Now I will connect to mutually coupled coils in series, what is the equivalent inductance that I can get? For this combination, current is entering the dot coil 1 and in coil 2 also it is entering the dot. What is the voltage induced in coil 1 $j\omega L_1 I_f + j\omega M I_f$, this is the voltage across in L_1 in coil 1. Voltage across in coil 2 $j\omega L_2 I_f + j\omega M I_f$. Finally the external voltage equation $v = I_f R + j\omega I_f (L_1 + M) + j\omega I_f (L_2 + M)$, plus voltage drop across coil 1 and voltage drop across coil 2. Voltage across coil 1 is $j\omega I_f L_1 + j\omega M I_f$. combine it we get the net inductance is $I_f R + j\omega I_f (L_1 + L_2 + 2M)$. (Refer slide time: 18:56 to 19:19)

Instead 'I' enters the dot in coil 1 and leaves the dot in coil 2

$$\therefore V = i_f R + j\omega i_f (L_1 + L_2 - 2M)$$

$$\therefore L_{eq} = L_1 + L_2 \pm 2M$$

When connected in parallel:

Assign i_1 & i_2 in Mesh 1 & 2


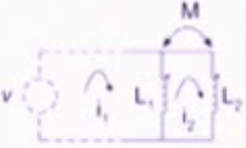
Current in Coil 1 = $(i_1 - i_2)$ enters the dot

Current in Coil 2 = i_2 enters the dot

\therefore 'V' in $L_1 = j\omega L_1 (i_1 - i_2) + j\omega M i_2$

'V' in $L_2 = j\omega L_2 i_2 - j\omega M (i_1 - i_2)$

\therefore 'V' = $jX_{L1} (i_1 - i_2) + j\omega M i_2 = jX_{L1} i_1 + j(-X_{L1} + X_{M2}) i_2$

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Slide 24 Coils connected in parallel

If I enters the dot in coil 1 and leaves the dot in coil 2, the voltage inducing coil 1 is $j\omega L_1 I_f - j\omega M I_f$, so $j\omega I_f (L_1 - M)$. (Refer slide time: 19:20 to 19:45)

Replacing series connected mutually coupled coils:

- Place the dots
- Current enters both the dots
- Sign of 'M' is the same as L

'V' in $L_1 = j\omega L_1 I_1 + j\omega M I_2$
 $= j\omega L_1 (L_1 + M)$

'V' in $L_2 = j\omega L_2 I_2 + j\omega M I_1$
 $= j\omega L_2 (L_2 + M)$

$\therefore V = I_1 R + j\omega L_1 (L_1 + M) + j\omega L_2 (L_2 + M)$
 $= I_1 R + j\omega L_1 (L_1 + L_2 + 2M)$
 $= I_1 R + j\omega L_{eq}$

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Slide 25 Place the dots

For the coil 2, $j\omega L_2 I_f - j\omega M I_f$, so $j\omega I_f (L_2 - M)$. (Refer slide time:19:46 to 20:13)

Instead 'I' enters the dot in coil 1 and leaves the dot in coil 2

$\therefore V = I_f R + j\omega I_f (L_1 + L_2 - 2M)$
 $\therefore L_{eq} = L_1 + L_2 - 2M$

When connected in parallel:

Assign i_1 & i_2 in Mesh 1 & 2

Current in Coil 1 = $(i_1 - i_2)$ enters the dot

Current in Coil 2 = i_2 enters the dot

\therefore 'V' in $L_1 = j\omega L_1 (i_1 - i_2) + j\omega M i_2$
 'V' in $L_2 = j\omega L_2 i_2 - j\omega M (i_1 - i_2)$
 \therefore 'V' = $jX_{L1} (i_1 - i_2) + j\omega M i_2 = jX_{L1} i_1 + j(-X_{L1} + X_{M1}) i_2$

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Slide 26 Equations

The net inductance is $I_f R + j\omega I_f (L_1 + L_2 - 2M)$. If I have two coils which are mutually coupled and if I connect them in series the two coupled values are

$(L_1+L_2\pm 2M)$, series adding, it is (L_1+L_2+2M) , if they oppose (L_1+L_2-2M) .

End of the lecture16